UNIT I REVIEW EXERCISES

This key gives answers to selected exercises only. Answers to some problems may not be given here.

ADDITIONAL EXERCISES FOR LESSON 1

I. Are simple propositions truth-functional? Why or why not?

Yes. A simple proposition has one component part, so its truth value is entirely dependent on that one part. In other words, the truth value of "God loves the world" clearly depends entirely on the truth value of "God loves the world."

2. Is a tautology a truth-functional proposition? Is a self-contradiction truth-functional? Explain your answers.

Neither tautologies nor self-contradictions are truth-functional, because they are true or false (respectively) by logical structure, which means that their truth values do not depend on the truth values of their component propositions. E.g., "$P \vee {\sim}P$", "$P \supset P$", and "$P \equiv {\sim}{\sim}P$" are true regardless of the truth value of P. Similarly, "$P \bullet {\sim}P$" is always false, regardless of the truth value of P.

3. Which of the following propositions are simple, and which are compound?

Something is rotten in the state of Denmark. SIMPLE

If it assume my noble father's person, I'll speak to it. COMPOUND

I did love you once. SIMPLE

I loved you not. COMPOUND (being the negation of the previous proposition)

The lady doth protest too much. SIMPLE

It is not nor it cannot come to good. COMPOUND

Rosencrantz and Guildenstern are dead. COMPOUND

ADDITIONAL EXERCISES FOR LESSON 2

I. Assume M means "All men are mortal." Give three distinct translations for ~M. Explain why "No men are mortal" is *not* a good translation of ~M.

Examples: "Not all men are mortal," "It is false that all men are mortal," and "Some men are not mortal." "No men are mortal" is a poor translation because it would mean that M and ~M could both be false (in the case that merely **some** men are mortal), but a proposition and its negation always have opposite truth values.

2. Assume T means "John is a teacher" and S means "John is a soldier." Give four translations for T • S. Give two translations for T ∨ S.

Examples: For T • S, "John is a teacher and John is a soldier," "John is a teacher and a soldier," "John is both a teacher and a soldier," and "John is a teacher, but he is also a soldier." For T ∨ S, "John is a teacher or John is a soldier," "John is a teacher or a soldier."

3. Give an example of a disjunction (a statement using *or*) that is best understood as an inclusive *or*. Give an example of a disjunction that is best understood as an exclusive *or*.

Examples: "Dinner comes with your choice of coffee or tea" (exclusive); and "You may have cream or sugar with your coffee" (inclusive).

4. Let the + sign indicate the logical operator "exclusive *or*" so that p + q means "p or q, but not both p and q." Complete a defining truth table for this logical operator. How then could the inclusive *or* be symbolized using this operator together with conjunction and negation?

p	q	p + q	p ∨ q	p + (~p • q)
T	T	F	T	T
T	F	T	T	T
F	T	T	T	T
F	F	F	F	F

5. Consider the defining truth table for conjunction. Notice that the conjunction of either truth value with a *true* (*T*) results in that same truth value (that is, $T \bullet T = T$, $F \bullet T = F$), and the conjunction of either truth value with a *false* (*F*) results in *F* (that is, $T \bullet F = F$, and $F \bullet F = F$). This can be stated more briefly with these two rules: $P \bullet T = P$, $P \bullet F = F$. How would you complete these rules for disjunction: $P \vee T = ?$ $P \vee F = ?$

$P \vee T = T$, $P \vee F = P$

6. Express the following compound propositions (from the book of Job, KJV) in symbolic form, using whatever propositional constants you think are reasonable:

That man was perfect and upright. (1:1)

P • U

In all this did not Job sin with his lips. (2:10)

~S

It shut not up the doors of my mother's womb, nor hid sorrow from mine eyes. (3:10)
~S • ~H

I was not in safety, neither had I rest, neither was I quiet; yet trouble came. (3:26)
(~S • ~R • ~Q) • T

It stood still, but I could not discern the form thereof. (4:16)
S • ~D

He shall lean upon his house, but it shall not stand: he shall hold it fast, but it shall not endure. (8:15)
(L • ~S) • (H • ~E)

With us are both the grayheaded and very aged men. (15:10)
G • A

Snares are round about thee, and sudden fear troubleth thee; or darkness . . . ; and abundance of waters cover thee. (22:10–11)
[(S • F) ∨ D] • W

For truly my words shall not be false. (36:4)
~~W (W = "My words shall be true)

ADDITIONAL EXERCISES FOR LESSON 3

Complete truth tables for the following compound proposition forms:

1. p ∨ ~q	2. ~(~p • q)	3. ~(p ∨ q)
T T F T	T FT F T	F T T T
T T T F	T FT F F	F T T F
F F F T	F TF T T	F F T T
F T T F	T TF F F	T F F F

4. p • (q ∨ ~r)	5. (p ∨ q) • (p ∨ r)
T T T T F T	T T T T T T T
T T T T T F	T T T T T T F
T F F F F T	T T F T T T T
T T F T T F	T T F T T T F
F F T T F T	F T T T F T T
F F T T T F	F T T F F F F
F F F F F T	F F F F F T T
F F F T T F	F F F F F F F

Assume that the propositions A and B are true, X and Y are false, and P and Q are unknown truth values. Determine the truth values (*true, false,* or *unknown*) of these compound propositions.

6. ~(A • X) TRUE

7. (Y • A) ∨ (B ∨ P) TRUE
8. ~(A • ~X) ∨ (~B • Y) FALSE
9. (P • ~P) ∨ Q UNKNOWN
10. ~[(P ∨ ~P) • ~(~Q • Q)] FALSE

ADDITIONAL EXERCISES FOR LESSON 4

1. Write your own *if* FALSE *then* TRUE conditionals and *if* FALSE *then* FALSE conditionals that appear to be true. Then write similar *if* FALSE *then* TRUE conditionals and *if* FALSE *then* FALSE conditionals that appear to be false. With the second set of conditionals, try putting them in the form ~(p • ~q). Do they still appear to be false?

   If a triangle is a square [F] then a triangle is a polygon [T] - appears true
   If a triangle is a square [F] then a triangle has four sides [F] - appears true
   If a triangle is a square [F] then a triangle has three sides [T] - appears false
   If a triangle is a square [F] then a triangle is a circle [F] - appears false
   It is false that both a triangle is a square and a triangle does not have three sides. - appears true
   It is false that both a triangle is a square and a triangle is not a circle. - appears true

2. Express the following compound propositions (from I Corinthians, KJV) in symbolic form, using whatever propositional constants you think are reasonable:

   If thou marry, thou hast not sinned; and if a virgin marry, she hath not sinned. (7:28)
   (M ⊃ ~S) • (V ⊃ ~I)
   Woe is unto me, if I preach not the gospel. (9:16)
   ~P ⊃ W
   When we are judged, we are chastened of the Lord. (11:32)
   J ⊃ C
   No man can say that Jesus is Lord, but by the Holy Ghost. (12:3)
   S ⊃ H
   Though I speak with the tongues of men and of angels, and have not charity, I am become as sounding brass, or a tinkling cymbal. (13:1)
   [(M • A) • ~C] ⊃ (B ∨ T)
   By [the gospel] also ye are saved, if ye keep in memory what I preached unto you, unless ye have believed in vain. (15:2)
   ~B ⊃ (K ⊃ S)

3. What is a necessary condition for passing a class? What is a sufficient condition for passing a class? Write these as standard *if/then* statements.

   A necessary condition for passing a class is something you must do in order to pass, though you may have to do other things in order to pass. "If you passed the class, then you must have attended class regularly." A sufficient condition is something that by itself is enough to pass. "If your final overall grade is greater than or equal to 70, then you will pass."

4. Write this as a standard *if/then* statement: "A person may be elected President of the United States only if that person has been a resident for fourteen years." Then use this conditional as the first premise of a *modus ponens* argument.

   "If a person is elected President of the US, then that person has been a resident for fourteen years. Theodore Roosevelt was elected President of the US. Thus he must have been a resident of the US for fourteen years."

5. Rewrite this conditional using the rule of transposition: "If there be no resurrection of the dead, then is Christ not risen" (I Cor. 15:13).

   If Christ is risen, then there is a resurrection of the dead.

6. Complete a truth table for the compound proposition ~p ⊃ p (this should require only two rows). What more basic proposition has the same truth table?

~p ⊃ p
FT T T
TF F F

   This has the same truth table as "p".

7. Consider the defining truth table for the conditional. How would you complete these rules: P ⊃ *T* = ? P ⊃ *F* = ? *T* ⊃ Q = ? *F* ⊃ Q = ? (*T* and *F* here don't stand for propositions; they stand for *true* and *false*.)

   P ⊃ *T* = *T*, P ⊃ *F* = ~P, *T* ⊃ Q = Q, *F* ⊃ Q = *T*

ADDITIONAL EXERCISES FOR LESSON 5

1. Complete the truth table for the compound proposition (p • q) ∨ (~p • ~q). What more basic compound proposition has the same truth table?

p	q	(p • q) ∨ (~p • ~q)	p ≡ q
T	T	T (T) F F F	T
T	F	F (F) F F T	F
F	T	F (F) T F F	F
F	F	F (T) T T T	T

2. Let B represent "The biconditional is true" and S represent "The truth values are the same." Symbolize the following propositions:

The biconditional is true if the truth values are the same. S ⊃ B

The biconditional is true only if the truth values are the same. B ⊃ S

The biconditional is true if and only if the truth values are the same. B ≡ S

ADDITIONAL EXERCISES FOR LESSON 6

1. Logic is based on three laws of thought. The Law of Identity states, "If a statement is true, then it is true." The Law of Excluded Middle states, "A statement is either true or false." The Law of Non-contradiction states, "A statement cannot be both true and false." These laws can be represented by three tautologies: p ⊃ p, p ∨ ~p, and ~(p • ~p), respectively. Show these to be tautologies by completing a truth table for each.

p	p ⊃ p	p ∨ ~p	~(p • ~p)
T	T (T) T	T (T) FT	(T) T F FT
F	F (T) F	F (T) TF	(T) F F TF

Determine whether the following pairs of propositions are *equivalent*, *contradictory*, or *neither*.

2. p ⊃ ~p, ~p ⊃ p CONTRADICTORY
3. p ∨ q, ~(~p ∨ ~q) NEITHER
4. p ≡ ~q, ~(p ≡ q) EQUIVALENT
5. p ⊃ (~p • q), ~p ⊃ (p • ~q) CONTRADICTORY
6. p • (q ∨ r), (p • q) ∨ r NEITHER
7. ~[(p • q) ∨ (p • r)], ~p ∨ (~q • ~r) EQUIVALENT

ADDITIONAL EXERCISES FOR LESSON 7

Use truth tables to determine the validity of the following brief (but unusual) arguments.

1. p ∴ q ⊃ q

p	∴ q ⊃ q
T	T T T
T	F T F
F	T T T
F	F T F

VALID

2. p ∴ q ⊃ p

p	∴ q ⊃ p
T	T T T
T	F T T
F	T F F
F	F T F

VALID

3. p q ∴ p ≡ q

p	q	∴ p ≡ q
T	T	T
T	F	F
F	T	F
F	F	T

VALID

Translate the following arguments into symbolic form, and then use truth tables to determine their validity.

4. You cannot have your cake and eat it too. Therefore you cannot have your cake.

~(H • E)	∴ ~H
F T T T	F
T T F F	F
T F F T	T
T F F F	T

←INVALID

5. I can tell the future. Thus it will either snow tomorrow, or it will not snow tomorrow.

F	∴ S ∨ ~S
T	T
T	T
F	T
F	T

←VALID

6. "But if there be no resurrection of the dead, then is Christ not risen. . . . But now Christ is risen from the dead. . . . So in Christ shall all be made alive" (I Cor. 15:13–22).

~R ⊃ ~C	C	∴ R
F T F	T	T
F T T	F	T
T F F	T	F
T T T	F	F

←VALID