

7. "Either your sister is telling lies, or she is mad, or she is telling the truth. You know she doesn't tell lies and it is obvious that she is not mad. . . . We must assume that she is telling the truth." (C. S. Lewis, *The Lion, The Witch, and The Wardrobe*)

$(L \vee M) \vee T$	$\sim L \bullet \sim M$	$\therefore T$
T	F	T
T	F	F
T	F	T
T	F	F
T	F	T
T	F	F
T	T	T
F	T	F

← VALID

8. "If the Son is not of the same substance with the Father, then He is a substance that was made, and if that substance was made, then all things were not made through Him. But, on the contrary, 'All things were made through him.' [Consequently], He is of the one and the same substance with the Father." (St. Augustine, *The Trinity*)

$\sim F \supset M$	$M \supset \sim A$	A	$\therefore F$
T	F	T	T
T	T	F	T
T	T	T	T
T	T	F	T
T	F	T	F
T	T	F	F
F	T	T	F
F	T	F	F

← VALID

ADDITIONAL EXERCISES FOR LESSON 8

Translate the following arguments into symbolic form, and then use shorter truth tables to determine their validity.

- I. Pluto is a planet if and only if it is reasonably large and has an independent, circular orbit around the sun. Pluto is reasonably large. Pluto has an independent, circular orbit around the sun. But Pluto's orbit lies outside the ecliptic plane. Therefore, it is false that if Pluto is a planet then its orbit does not lie outside the ecliptic plane.

$P \equiv (L \bullet O)$	L	O	E	$\sim(P \supset \sim E)$
F T T T T	T	T	T	F F T F T
↑				VALID

2. If the spirit of Samuel was raised by the witch at Endor, then spirits of dead men could be seen by the living. Ghosts at one time existed if spirits of dead men could be seen by the living. Ghosts exist now only if ghosts at one time existed. So either the spirit of Samuel was not raised by the witch at Endor, or ghosts exist now.

$R \supset S$	$S \supset F$	$N \supset F$	$\therefore \sim R \vee N$
T T T	T T T	F T T	F T F F
			INVALID

3. A murder is committed if and only if an innocent person is killed and killed with either malice aforethought or the demonstration of an abandoned or malignant heart. If a fetus is aborted, then an innocent person is killed, without malice aforethought, but with a demonstration of an abandoned or malignant heart. Consequently, if a fetus is aborted, then a murder is committed.

$M \equiv [K \bullet (A \vee D)]$	$F \supset [K \bullet (\sim A \bullet D)]$	$\therefore F \supset M$
F T T T F T T	T T T T T F T T	T F F
↑		VALID

4. If God exists, then He is both perfectly good and infinitely powerful. If God is perfectly good then He always desires to prevent evil. If God is infinitely powerful then He is always able to prevent evil. If evil exists, then either God does not always desire to prevent evil or He is not always able to prevent evil. Evil does exist. Therefore God does not exist.

$G \supset (P \bullet I)$	$P \supset D$	$I \supset A$	$E \supset (\sim D \vee \sim A)$	E	$\therefore \sim G$
T T T T T	T T T	T T T	T T F F F	T	F
			↑		VALID

5. If the last argument was valid, then if the conclusion was false, then one of the premises must be false. The conclusion was false. If you did the last problem correctly, then the argument was valid. Thus, either you did not do the last problem correctly or one of the premises must be false.

$V \supset (C \supset P)$	C	$D \supset V$	$\therefore \sim D \vee P$
T T T F F	T	T T T	F F F
↑			VALID

ADDITIONAL EXERCISES FOR LESSON 9

1. Write your own valid argument, different from any in the text, which requires you to guess a truth value to determine the validity by the shorter truth table.

$$\frac{p \equiv (q \vee r) \quad q \supset r \quad \therefore p \equiv r}{T \quad T \quad F}$$

(Example: The conclusion can be false in two ways.)

Translate the following arguments into symbolic form, and then determine their validity using the shorter truth table method.

2.

flowing. Either the diode is not lit or current is flowing. Consequently, the switch is closed if and only if the diode is lit.

$$\frac{C \supset L \quad C \vee \sim F \quad \sim L \vee F \quad \therefore C \equiv L}{TTF \quad TT \quad T \quad \underline{IFF} \quad \text{(guesses underlined)}} \\ \uparrow \\ FFT \quad FTF \quad FTT \quad EFT \quad \text{VALID} \\ \uparrow$$

3. If you can do judo and karate then you can defend yourself. You can do judo or karate. Therefore you can defend yourself.

$$\frac{(J \bullet K) \supset D \quad J \vee K \quad \therefore D}{TFFTF \quad TTF \quad F} \quad \text{INVALID}$$

4. If this argument is valid, then you can prove it to be valid if and only if you can use a truth table. Either this argument is valid and you can prove it to be valid, or this argument is valid and you can use a truth table. Therefore, you can prove this argument to be valid and you can use a truth table.

$$\frac{V \supset (P \equiv T) \quad (V \bullet P) \vee (V \bullet T) \quad \therefore P \bullet T}{FTTFF \quad FFTTFFF \quad \underline{IFF}} \\ \uparrow \\ TTFFT \quad TFFTTT \quad EF \quad \text{VALID} \\ \uparrow$$

ADDITIONAL EXERCISES FOR LESSON 10

Determine the consistency of these sets of propositions using shorter truth tables.

$$1. \frac{p \bullet (q \vee r) \quad q \bullet \sim r \quad p \vee r}{TT \quad TTF \quad TTT \quad TTF} \quad \text{CONSISTENT}$$

$$2. \frac{\sim p \vee (q \bullet r) \quad p \bullet \sim r}{F \quad T \quad FF \quad TTT}$$

↑

INCONSISTENT

$$3. \frac{\sim(p \vee q) \quad \sim p \supset (q \vee r) \quad \sim r}{T \quad FFF \quad T \quad T \quad FFF \quad T}$$

↑

INCONSISTENT

4. If you do not have your own business then if you want to make money then you must be an employee. You must be an employee. You have your own business and you want to make money.

$$\frac{\sim B \supset (M \supset E) \quad E \quad B \bullet M}{F \quad T \quad TTT \quad T \quad TTT} \quad \text{CONSISTENT}$$

5. Either Lefty or Capone is responsible for the fire. If Capone is responsible for the fire, then he knew about the dangerous conditions but did nothing about them. Capone knew about the dangerous conditions only if he did something about them. Lefty is not responsible for the fire.

$$\frac{L \vee C \quad C \supset (K \bullet \sim D) \quad K \supset D \quad \sim L}{FTT \quad TTTT \quad TTF \quad T}$$

↑

INCONSISTENT

ADDITIONAL EXERCISES FOR LESSON 11

1. Two propositions are logically equivalent if they imply each other. Explain how the shorter truth table method determines whether or not the propositions imply each other.

A shorter truth table for equivalence assumes that the first statement does not imply the second by assigning the first to be true and the second false, then assumes that the second statement does not imply the first by assigning the second to be true and the first to be false. It then looks for contradictions to these assumptions. In other words, the method is the same as shorter truth tables for validity in both directions.

Demonstrate the following propositions to be equivalent using both the longer and the shorter truth table methods. In each case, which is the better method, and why?

2. $[p \bullet (p \vee q)] \equiv [p \vee (p \bullet q)]$

p	q	$p \bullet (p \vee q)$	$p \vee (p \bullet q)$
T	T	T	T
T	F	T	F
F	T	F	T
F	F	F	F

EQUIVALENT

The longer truth table may be easier, though longer, because the shorter truth table requires you to guess a truth value, which is prone to errors.

3. $[\sim(p \equiv q)] \equiv [(p \vee q) \bullet \sim(p \bullet q)]$

p	q	$\sim(p \equiv q)$	$(p \vee q) \bullet \sim(p \bullet q)$
F	T	T	T
T	F	T	F
T	T	F	F
F	F	F	F

EQUIVALENT

The longer truth table is much easier, because you must guess both ways for the shorter truth table. The "shorter" truth table is not really shorter here.

4. $[(p \supset q) \bullet r] \equiv [\sim(r \supset p) \vee (q \bullet r)]$

p	q	r	$(p \supset q) \bullet r$	$\sim(r \supset p) \vee (q \bullet r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	F	T
F	T	F	F	F
F	F	T	F	T
F	F	F	F	F

EQUIVALENT

Though it requires one to guess, the shorter truth table is easier in this case.

Translate the following pairs of propositions into symbolic form, and then use the shorter truth table method to determine their equivalence.

5. If an animal can learn by trial and error and recognize words, then that animal can reason. If an animal can recognize words, then it is false that the animal can learn by trial and error but is not able to reason.

$(L \bullet W) \supset R$	$W \supset \sim(L \bullet \sim R)$	
T T T T F	T F F T T T F	
↑		
T T T F F	T T F T T T F	EQUIVALENT
	↑	

6. Pirates murder, and they either rape or pillage. Pirates rape and murder, or they pillage.

$M \bullet (R \vee P)$	$(R \bullet M) \vee P$	
T T F T F	F F T F F	
↑		
T F F F F	F F T T F	
	↑	
F F T T	F F T T	NOT EQUIVALENT

7. My grandfather was a soldier or a sailor, and he was not a soldier. My grandfather was a sailor.

$(O \vee A) \bullet \sim O$	A	
F T T T T F	F	
↑		
T T T F F T	T	NOT EQUIVALENT

ADDITIONAL EXERCISES FOR LESSON 12

1. In Matthew 22:15–22, Jesus was presented with a dilemma by the Pharisees and the Herodians. Write out the full dilemma. What method does Jesus use to refute it?

One possibility for the full dilemma: "If Jesus says it is lawful to pay taxes to Caesar, then he admits the legitimacy of Caesar's authority and the people will oppose him. If Jesus says it is not lawful to pay taxes to Caesar, then he denies the legitimacy of Caesar's authority and the civil rulers will oppose him. Jesus will either say it is lawful to pay taxes to Caesar or it is not lawful to pay taxes to Caesar. There-

fore, he either admits the legitimacy of Caesar's authority and the people will oppose him, or he denies the legitimacy of Caesar's authority and the civil rulers will oppose him." Jesus appears to go between the horns, admitting the legitimacy of Caesar's authority to a point: Caesar may demand our money (which has his image on it), but he may not demand our worship (which belongs to God alone).

2. In Matthew 21:23–27, Jesus refutes a challenge brought to him by the chief priests and elders. Explain how this may be an example of rebutting the horns of a dilemma.

Given the response of Jesus, the Jews appeared to be assuming a dilemma such as this: "If Jesus says his authority is from heaven, then we can accuse him of blasphemy. If Jesus says nothing, then he admits our authority to silence him. Jesus will either say his authority is from heaven, or he will say nothing. Therefore we will either accuse him of blasphemy, or we will gain authority to silence him." Jesus rebuts the horns by producing a similar counter-dilemma: Is John's baptism from heaven or from men? Had they answered this question honestly, they would have answered their own question.

3. One year, a lazy farmer refused to work in his field, reasoning this way: "If God knows that I will have a crop, then it will not be necessary for me to work to produce one. If God knows that I will not have a crop, any work of mine will be ultimately wasted. God knows whether or not I will have a crop. So my work would be either unnecessary or wasted." Refute this dilemma.

Grasp the horns: God knows that you will have a crop by working to produce one.

4. One classic dilemma is the legend of Caliph Omar's destruction of the Library of Alexandria. Omar is reported to have said, "If the books of this library merely repeat the Koran, then they are superfluous. If they disagree with the Koran, then they are wicked. They either repeat the Koran or disagree with it. In either case they are superfluous or wicked, and should be destroyed." Refute this dilemma using all three methods.

Grasp the horns: "Even if these books repeat what is in the Koran, they are not necessarily superfluous. Wouldn't you agree that more repeating of the Koran is better? Or if they disagree with the Koran, they are not necessarily wicked, not if the Koran is in error" (though this would likely not be persuasive).

Go between the horns: "The choices are not only repeating the Koran or disagreeing with it. These books may be consistent with the Koran without repeating it." Rebut the horns: "When you speak, Mr. Omar, if you merely repeat the Koran, then your words are superfluous, or if you disagree with the Koran, then you are wicked. You either repeat the Koran or you disagree with it, even as these books. Therefore, you are either superfluous or wicked, and should be destroyed."